Search for periodicities in experimental data using an autoregression data model

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Abstract

To process data obtained during interference experiments in high-energy physics, methods of spectral analysis are employed. Methods of spectral analysis, in which an autoregression model of experimental data is used, such as the maximum entropy technique as well as Pisarenko and Prony's method, are described. To show the potentials of the methods, experimental and simulated hummed data are discussed as an example.

An alternative for an increase in the energy of colliding particles, observed when studying the properties of a substance at short distances, is the observation and study of the fine effects of the bound states of atoms and elementary particles [1]. Of special interest are the interference energies and phases of states sensitive to the behaviour of the potential at short distances and to external effects [2]. Information on the bound states of systems is obtained in interference experiments [3]. The data obtained during such an experiment are represented by oscillating relations the spectral parameters of which are assessed to cast light on the energy and phase characteristics of a system [4]. The reliability of the resultant information depends on the ability to distinguish periodicities in experimental data and to estimate their parameters.

The mathematical theory and algorithmic apparatus, used to reveal latent periodicities, have an extensive background and various practical applications. Methods for identification of periodicities are subdivided into a) approximation methods that enable a person to approximate experimental data by a function which a priori agrees with the preset model and b) filtration methods that provide information on components and their parameters when a priori evidence is minimum. The spectral characteristics of a signal can be estimated more accurately by approximation methods than by filtration methods [5].

The authors discuss a search for latent periodicities in differently generated data using methods based on an autoregression data model (ARDM) [6].

ARDM-based methods have an intermediate position between the groups of data processing methods discussed. In these methods, approximation is represented by a postulate of local data connection and filtration by operations done to produce a spectrum and to identify components. Owing to their intermediate position, ARDM methods take the advantages of each of the groups described. Spectral parameters and resolution can be estimated more accurately by ARDM methods than by filtration methods on the basis of Fouriers discrete transformation (FDT) [7, 8].

Unlike FDT algorithms in which the signal observed is split into harmonic components, some of ARDM methods (e.g. Prony's method) use decomposition into components in the form of damping oscillations. When processing interference experimental data, such a representation has been found to be more realistic than a conventional one because it

considers the lifetime of states. This tendency persisted in Wavelet analysis, where decomposition into soliton-like components made it possible to follow space-time variations in the spectral characteristics of signals [9].

Oscillation phases can be estimated more accurately by Prony's method than by FDT method. This advantage is important, considering the informative capacity of phases in interference experiments.

ARDM modelling was done in the present study using the spectral maximum entropy data technique (MENT), Pisarenko's method and Prony's method [10–12], simulated and experimental data being used as an example.

Autoregression data model. ARDM of a random process postulates a local data connection: the current value of the function $y(l)$, which approximates the process studied, is estimated from the previous data counts $x(l-k)$ in the form of a linear combination:

$$y(l) = \sum_{k=1}^{p} a(k)x(l-k) \tag{1}$$

with depth p and parameters $a(k)$. The parameters of $a(k)$ contain all information on the spectral characteristics of the process [13].

Deducting the value of the variable $x(l)$ from both parts of equality (1), the error in the current value $\varepsilon(l) = x(l)$ is expressed as:

$$\varepsilon(l) = \sum_{k=0}^{p} a(k)x(l-k) \tag{2}$$

with the parameter $a(0) = -1$. Because the error in the current value is also represented as a linear combination of previous data counts, the parameters of $a(k)$ can be regarded as the coefficients of a linear filter which predicts an error.

An error in predicting the current error value can be minimized, for example, on the basis of the least squares principle. In the case of an independent signal and additive noise, the observed data supplied to the input of the error prediction filter are converted to white noise. It is an example of a so-called "bleaching" filter.

Multiplying both parts of relation (2) by the complex conjugate value $x^*(l)$ and averaging for all observed data, we will get a system of linear equations (3) for the parameters of $a(k)$:

$$\sum_{k=0}^{p} R(m-k)a(k) = \sigma^2\delta(m) \tag{3}$$

where σ^2 is noise dispersion, $\delta(m)$ is delta-function, $R(m-k)$ is the autocorrelation function values of $R(k)$, $m = 0, 1, 2 \ldots p$.

To determine the filter coefficients of $a(k)$, Derbu-Levinson's algorithm [14] is commonly used by consecutively estimating bleaching filter coefficients more accurately and calculating the power of the error predicted, beginning with a first-order filter and ending with p-th-order filter. The algorithm calculates the sequence (a_{11}, σ_1^2), $(a_{21}, a_{22}, \sigma_2^2), \ldots,$ $(a_{p1}, a_{p2}, \ldots, a_{pp}, \sigma_p^2)$ using the formulas:

$$a_{11} = -R(1)/R(0); \quad \sigma_1^2 = (1 - |a_{11}|^2)R(0)$$

$$a_{kk} = -\left[R(k) + \sum_{i=1}^{k-1} a_{k-1,i} R(k-i)\right] / \sigma_{k-1}^2 \tag{4}$$

$$a_{ki} = a_{k-1,i} + a_{kk} a_{k-1,k-i}^*; \quad \sigma_k^2 = (1 - |a_{kk}|^2)\sigma_{k-1}^2$$

The rapidity and economy of Derbu-Levinson's algorithm are due to the fact that, unlike the algorithm used to solve systems of Gauss linear equation, it requires p^2 operations instead of p^3 operations and uses the results of previous calculations. More efficient algorithms have been developed for the same purposes [15].

Maximum entropy technique (MENT). As a spectral method, it was first proposed by Burg [10] to maximize the entropy density functional of the spectral capacity of a process $P(l)$

$$-\sum_{l=-(n-1)/2}^{(n-1)/2} \ln P(l) \to \max \tag{5}$$

when fulfilling the conditions of Winer-Hinchin's theorem for $p+1$ known values of the autocorrelation function

$$\sum_{l=-(N-1)/2}^{(N-1)/2} P(l) \exp(-j\Delta\omega lk\Delta t) = R(k), \tag{6}$$

where $\Delta\omega$ and Δt are frequency and time intervals between spectrum and data counts, j is imaginary unit ($j^2 = -1$), $k = 0, 1, 2, \ldots p$.

For the Gaussian random process, the requirement of a maximum entropy functional is equivalent to the minimum functional of a predicted error. This variation problem with the bleaching filter coefficients of $a(k)$ as Lagrange factors is solved using the formula:

$$P(l) = \frac{\sigma^2}{\left|1 + \sum_{k=1}^{p} a(k) \exp(-j\Delta\omega lk\Delta t)\right|^2} \tag{7}$$

which has a simple meaning: the power spectrum of a signal is found by dividing the output power of noise by the squared module of the spectral characteristics of the bleaching filter. The coefficients of $a(k)$ are estimated in ARDM by solving a system of equations (3).

MENT considers local data connection, estimates the power spectrum of noise, does not result in negative values in the power spectrum and has a better resolution as compared with FDT, but is inferior in resolution to Pisarenko's and Prony's methods. As MENT is equivalent to the least squares method, it gives an undisplaced estimate of the spectrum. The requirement of maximum entropy or minimum information at preset restrictions automatically excludes all alien frequency peaks from the estimate of the spectrum. For this reason, the spectral peaks in MEM that correspond to an anharmonic periodic histogram are expected to be weaker than in FDT [16].

MENT is efficient for express estimation of the frequency spectrum of random processes. It can also be employed to reveal latent periodicities. To show this, let us discuss the dependence of the ionization potential of an atom in basic state on the atomic number

of an element [17] (Fig.1a) and the time dependence of the rate of water flow through the Solomensky Strait of the Petrozavodsk Bay in Lake Onega [18] (Fig. 2a). To produce frequency spectra, programme [19] was used in these examples.

The frequency spectrum in Figure 1b shows periods 8, 10, 18 and 32 characteristic of Mendeleyev's Table. The FDT spectrum of such an anharmonic distribution contains, in addition to the main periods $T_{k,l}$, the periods $T_{k,i} = T_{k,i-1}\dfrac{i-1}{i}$ equal to 4.0, 2.7 and 2 for the most intensive and narrow peak 8. In this spectrum, poorly intensive peaks are far more numerous, and among the periods 6.0, 4.7, 4.1, 3.1, 2.8, 2.3, 2.2 and 2.1 only 4.1, 2.8 and 2.1 can be considered to be close to anharmonic periods.

In the second example, the pattern of the signal studied is fairly close to a combination of harmonics and a noise component. In the power spectrum of this signal (Fig. 2b), all medium- and high-frequency peaks were interpreted. These peaks were correlated in the order of increasing periods with the seiches oscillations of the water body connected through the strait with the Petrozavodsk Bay, those of the Petrozavodsk Bay and those of Great Lake Onega. Correlation with observed field data and simulation data gave consistent results [18]. Spectral analysis revealed a ca. 12 hour tidal period, unknown earlier for lakes, and other low-frequency periods that were hard to interpret. The spectral resolution of MEM proved to be quite sufficient for analysis of the results obtained.

Pisarenko's and Prony's methods. Pisarenko's method, used to distinguish harmonic components from their combination with white noise, provides even higher spectral resolution than MENT. It became possible to increase spectral resolution in Pisarenko's method at least 1000 times as compared with that of MENT. In this method, the frequencies of components are calculated, whereas in FDT or MENT they are estimated visually from the positions of the peaks of the frequency spectrum. Frequency estimation accuracy and spectral resolution are preset by the accuracy of calculations.

To show the algorithm used to estimate frequencies in Pisarenko's method, let us discuss the sinusoid $x(l) = sin(\Omega l)$. The trigonometric identity

$$\sin(\Omega l) = 2\cos\Omega \cdot \sin(\Omega(l-1)) - \sin(\Omega(l-2)) \tag{8}$$

provides a link between data:

$$x(l) = 2\cos\Omega \cdot x(l-1) - x(l-2) \tag{9}$$

The Fourier's transformation of relation (9) results in the relation:

$$X(\omega) \cdot \left(1 - 2\cos\Omega \cdot z^{-1} + z^{-2}\right) = 0, \tag{10}$$

where z is understood as $\exp(-j\Omega)$. The roots of the equation of second degree $z_1 = z_2^*$ determine the frequencies Ω and $-\Omega$ in accordance with the expression

$$\Omega = \mathrm{arctg}(\mathrm{Im}z_i/\mathrm{Re}z_i). \tag{11}$$

In a general case, local data connection in Pisarenko's method is preset in the form:

$$x(l) = -\sum_{k=1}^{2p} a(k)y(l-k) + \varepsilon(l), \tag{12}$$

and for the coefficients of $a(k)$ a system of linear equations:

$$\sum_{k=0}^{2p} R(m-k)a(k) = \sigma^2 a(m) \tag{13}$$

is obtained.

The coefficients $a(k)$ and noise dispersion σ^2 are estimated by calculating the eigen numbers and eigen vectors of the matrix of autocorrelation functions $R(m,k)$ [20]. Dispersion σ^2 corresponds to the minimum eigen number of the matrix $R(m,k)$. In this case, it is convenient to use the recursive expression $R\vec{c}(l+1) = \vec{c}(l)$, which follows from (13), and to determine the vector $\vec{c}(l+1)$ from the vector $\vec{c}(l)$ estimated from previous iteration. It is convenient to take the vector $\vec{c}(0) = [1,1,1,...,1]$ for initial approximation. Several iterations make it possible to produce a vector close to $\vec{c}(\infty)$. It is used to estimate $\lambda_{\min} = \sigma^2 = \dfrac{\vec{c}^T R\vec{c}}{\vec{c}^T\vec{c}}$ and vector $\vec{a}$ is determined as $\vec{a} = \dfrac{\vec{c}}{\lambda_{\min}}$.

The structure of the matrix of the autocorrelation functions $R(m,k)$ is such that if harmonics are present in the process studied, then the filter coefficients $a(k)$ are valid and meet the requirement: $a(k) = a(2p-k)$. Composing and solving an algebraic equation of $2p$ degree with the real coefficients $a(k)$ and $a(0) = 1$

$$\sum_{k=0}^{2p} a(k) z^{2p-k} = 0, \tag{14}$$

complex conjugate roots, equal in module to unit, are calculated. The roots determine the frequencies of harmonics in accordance with expression (11).

The positive frequencies of harmonics Ω_i, $i = 1, 2, \ldots, p$, are used to estimate spectral power frequencies $P(i)$, solving a system of equations

$$\sum_{i=1}^{p} P(i)\cos(\Omega_i l \Delta t) = R(l). \tag{15}$$

The self-consistency of the results obtained is checked by estimating noise dispersion σ^2 and expressing it by expression:

$$\sigma^2 = R(0) - \sum_{k=1}^{p} P(i). \tag{16}$$

Using Pisarenko's method, one can estimate the dispersion of additive white noise. However, obtaining non-negative estimates of the spectral density of harmonics is not guaranteed. Like in MENT, the initial phase of harmonics is not determined in Pisarenko's method.

This shortcoming is overcome in Prony's method in which Pisarenko's idea is generalized for non-steady periodic processes, using damping oscillations as a basis for spectral decomposition.

In Prony's method, the function $y(l)$, which approximates the process, is expressed through the complex numbers b_i and z_i by the expression

$$y(l) = \sum_{i=1}^{2p} b_i \cdot z_i, \quad l = 0, 1, 2, ..., n-1, \tag{17}$$

$$b_i = A_i \exp(j\theta_i)$$
$$z_i = \exp(\alpha_i + j\Omega_i \Delta t)$$

where the parameters A_i, α_i, Ω_i and θ_i are the amplitude, coefficient of damping, frequency and initial phase of the i-th component of the decomposition of reference data.

Local connection between data is postulated like in Pisarenko's method, the coefficients $a(k)$ are also calculated and used to compose polynomial (14). The difference lies in the fact that the roots of the polynomial are now represented by complex exponents z_i.

Proceeding from the minimum least squares functional

$$\sum_{l=0}^{n-1} (x(l) - y(l))^2 \to \min, \tag{18}$$

the vector $\vec{B} = (b_1, b_2, b_3, \ldots, b_{2p})$, which is dependent on the relation

$$\vec{B} = (F^{\#}F)^{-1} F^{\#} \vec{X}, \tag{19}$$

is estimated. In the above relation, $\vec{X}$ is a data line, F is Van der Mond's matrix composed of the degrees of roots z_i, $F^{\#}$ is a transposed matrix. From this relation and the roots of polynomial z_i the parameters of decomposition components are estimated:

$$A_i = |b_i|; \quad \Omega_i = \operatorname{arctg}(\operatorname{Im} z_i / \operatorname{Re} z_i)$$
$$\alpha_i = (\ln |z_i|)/\Delta t; \quad \theta_i = \operatorname{arctg}(\operatorname{Im} b_i / \operatorname{Re} b_i) \tag{20}$$

Finally, Prony's method gives a data estimate:

$$s(t) = \sum_{i=1}^{2p} A_i \exp(\alpha_i t) \cdot \exp(j(\omega_i t + \theta_i)), \tag{21}$$

its Fourier image:

$$S(\omega) = \sum_{i=1}^{2p} A_i \exp(j\theta_i) \frac{1}{(\omega - \omega_i) - j\alpha_i} \tag{22}$$

and an energy spectrum:

$$E(\omega) = |S(\omega)|^2. \tag{23}$$

It has already been noted that an advantage of Pronys method is a broader class of functions used as a basis of spectral decomposition. The formalism of the method allows the incorporation of exponentially falling and rising signals into this class. Therefore, methods of spectral analysis can be applied to monotonically varying and transitional random processes. The opportunity to calculate the amplitudes, coefficient of damping, frequency and especially phase of oscillations can be used to interpolate and extrapolate signals. In some cases, this helps forecast processes, which is a difficult problem.

To simulate Pronys method, the authors wrote a programme in Fortran. Figure 3 shows initial signals in the form of a single sinusoid (a) and a mixture of four different

harmonics with additive white noise (b) and the results of their reconstruction based on the parameters estimated by decomposition. For the sinusoid preset by the formula

$$1.600\cos(2.021t + 1.0798) \tag{24}$$

with 15% additive noise the estimate

$$1.594\cos(2.021t + 1.0930) \tag{25}$$

was calculated, and for a mixture of harmonics

$$\begin{aligned} &2.000\cos(0.5054t + 0.2670) + 2.000\cos(1.015t + 0.5422) + \\ &+ 1.000\cos(1.844t + 0.948) + 1.600\cos(2..465t + 1.3167) \end{aligned} \tag{26}$$

with 6% noise the estimate

$$\begin{aligned} &1.932\cos(0.5054t + 0.2821) + 1.957\cos(1.015t + 0.5887) + \\ &+ 0.9216\cos(1.844t + 0.9716) + 1.636\cos(2.465t + 1.2530) \end{aligned} \tag{27}$$

The above results are in good agreement with the reference signals. It has been shown in these and other computing experiments that Prony's method estimates frequencies most accurately and the phases of components and the coefficients of damping of components least accurately and that the spectral parameters of high-frequency components are more accurate than those of low-frequency components.

In the ARDM methods, a priori information, preset in the form of local data connection, is generalized, can be used to form a model of an oscillating signal and to determine its composition and the spectral parameters of components without any special a priori assumptions. The resultant parameters can be estimated more accurately within the model formed (in some cases, it is desirable), using other powerful approximation methods, e.g. the Monte Carlo method [21].

Conclusions:

1. Methods of spectral analysis are highly informative. They are employed in interference experiments to cast light on the characteristics of the bound state of systems, such as atoms, nuclei and particles.

2. ARDM-based methods, used to estimate the spectral characteristics of signals, combine the advantages of approximation and filtration methods.

3. The estimates obtained by MENT for the power spectra of real signals are informative, and can be used to analyse harmonic signals and anharmonic distributions. The spectral parameters of oscillating signals, estimated using Prony's method, are highly accurate and allow to represent experimental data by the superposition of basic functions.

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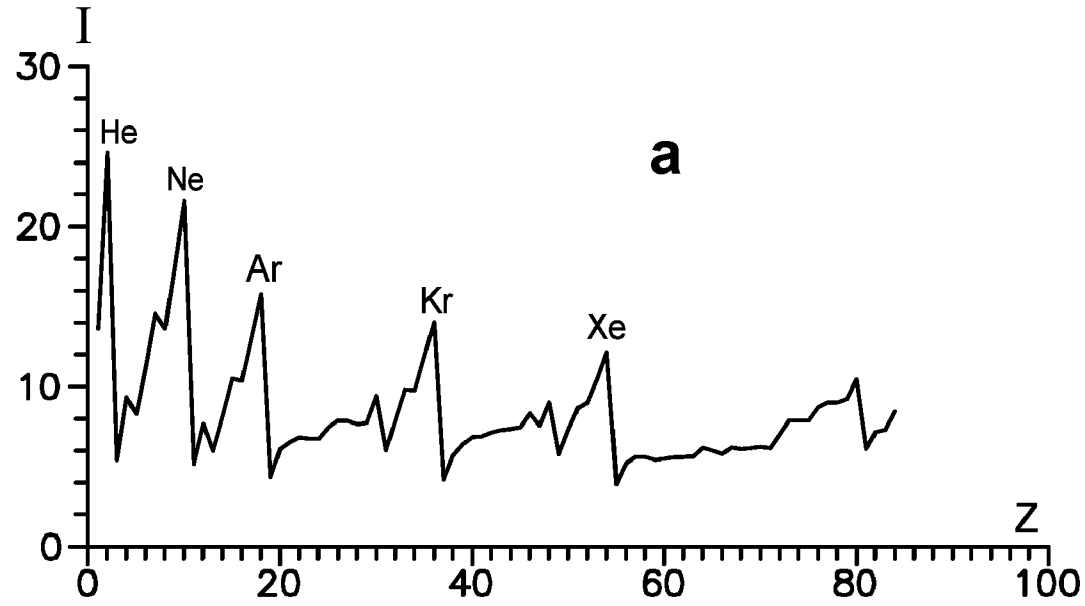
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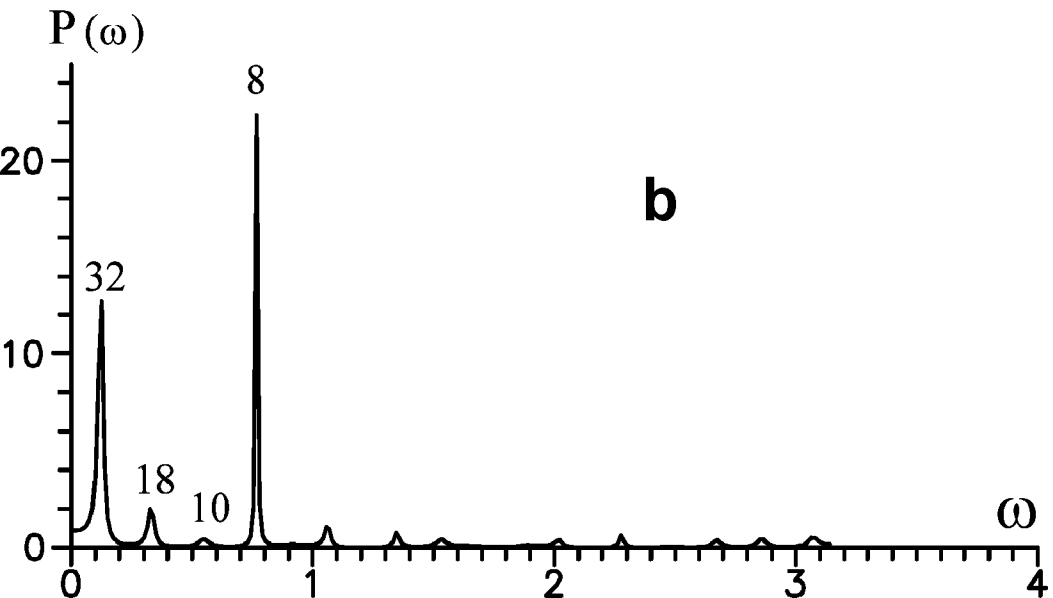
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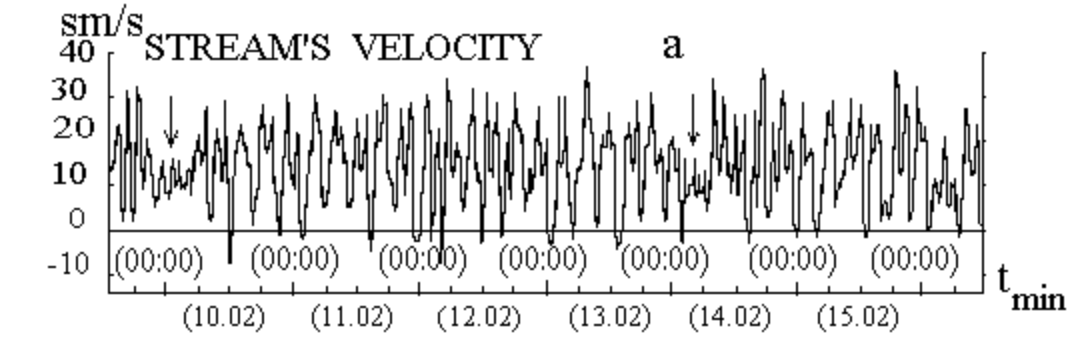
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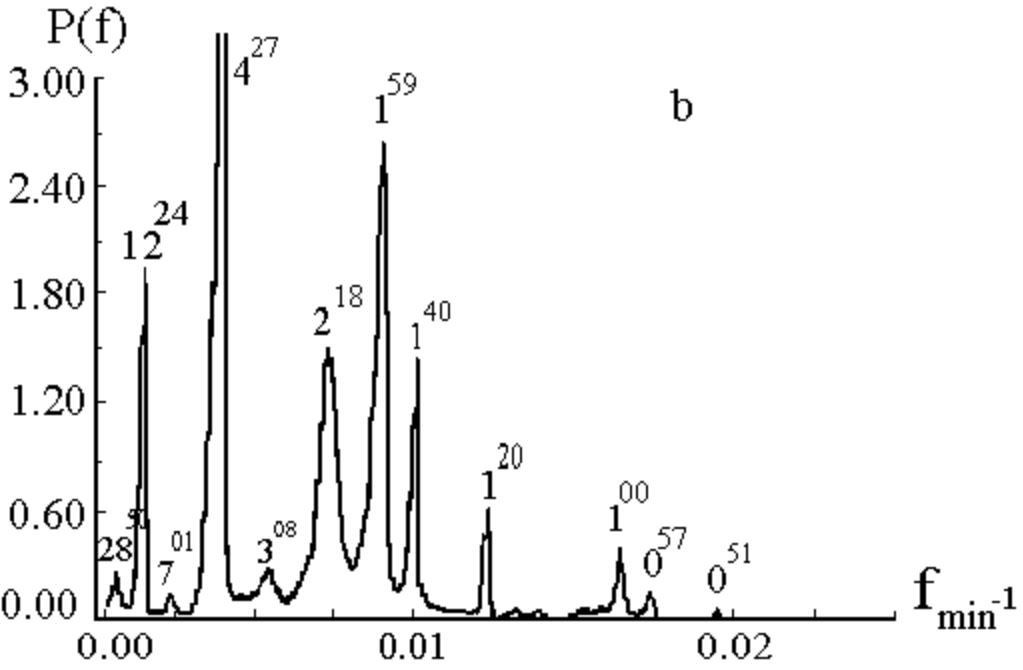
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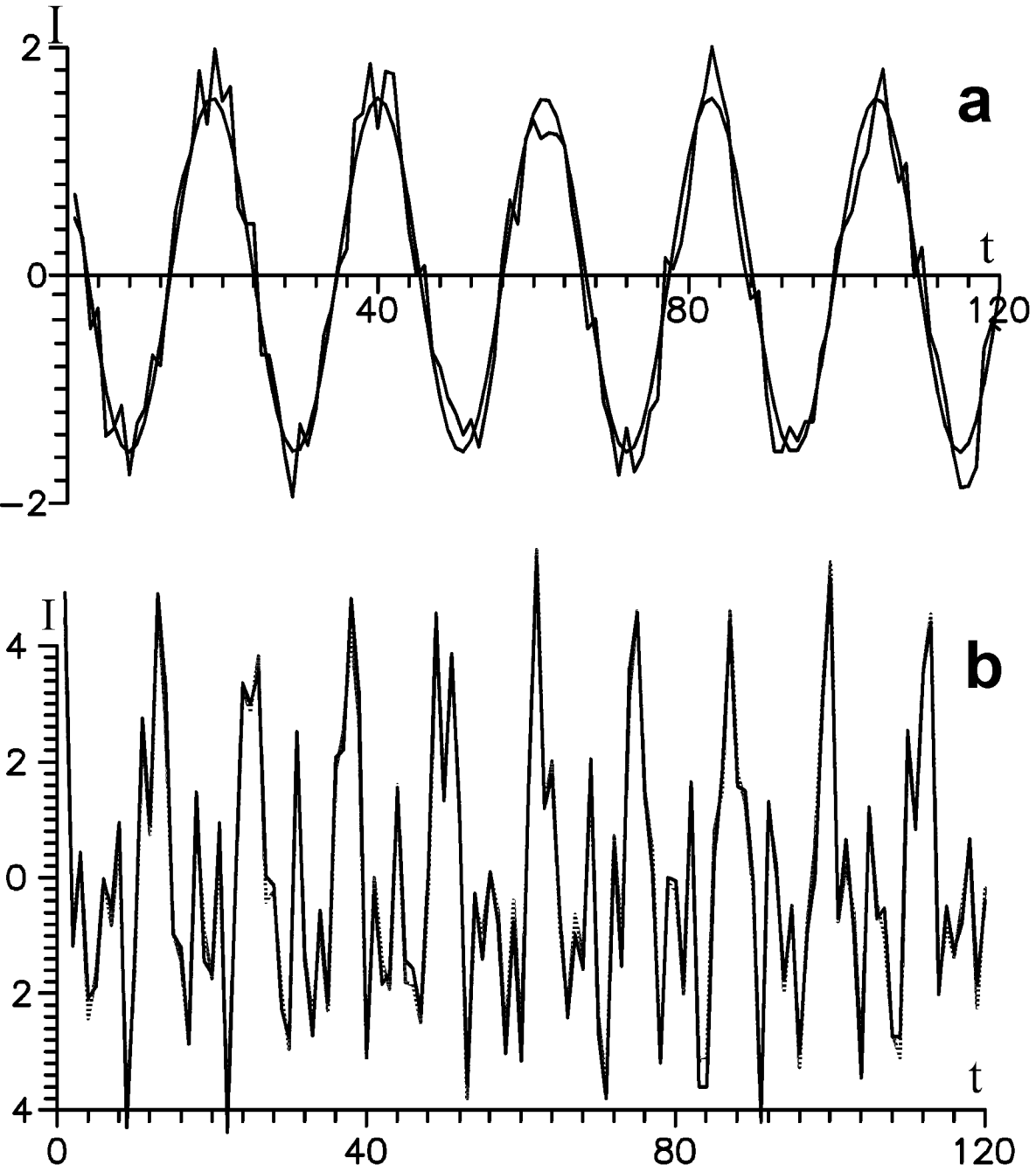
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